Problem Set #3: Solutions

1. (a) Mass of gas swept up = particle area × velocity × time × gas density.

$$M = \pi s^2 \times \eta V_K \times P \times \rho$$

orbital period $P = \dfrac{2\pi a}{V_K}$

$$\Rightarrow \quad M = \pi s^2 \cdot \eta V_K \cdot \frac{2\pi a}{V_K} \cdot \rho$$

$$M = 2\pi^2 \eta \cancel{V_K} s^2 a \rho$$

$$\underline{\underline{M = 2\pi^2 \eta s^2 a \rho}}$$

(b) Particle's own mass $M = \frac{4}{3}\pi s^3 \rho_M$

Equating with above expression

$$\tfrac{4}{3}\pi s^3 \rho_M = 2\pi^2 \eta s^2 a \rho$$

$$\Rightarrow \quad s = \tfrac{3}{2}\pi \eta a \left(\frac{\rho}{\rho_M}\right)$$

Substituting: $\underline{\underline{s \simeq 3.5 \times 10^2 \text{ cm}}}$

2. Comet parameters : $P = 10^6$ yr

$r_p = 1$ AU

$s = 5$ km

$\rho_n = 1 \text{ g cm}^{-3}$

(a) From Kepler's laws : $P_{yr}^2 = a_{AU}^3$

$$a = 10^4 \text{ AU}$$

Perihelion distance $r_p = a(1-e)$

$$1 - e = r_p/a$$

$$e = 1 - r_p/a = 0.9999$$

Aphelion distance $r_a = a(1+e)$

$$= 2 \times 10^4 \text{ AU}.$$

(b) Energy conservation implies:

$$\frac{1}{2} v^2 - \frac{GM}{r} = -\frac{GM}{2a}$$

$$\frac{1}{2} v^2 = GM\left(\frac{1}{r} - \frac{1}{2a}\right)$$

$$v = \left[2GM\left(\frac{1}{r} - \frac{1}{2a}\right)\right]^{1/2}$$

@ 1 AU $\quad \underline{\underline{v = 4.21 \times 10^6 \text{ cm s}^{-1}}}$

(c) Earth's orbital velocity $V_e = \sqrt{\frac{GM}{a}}$

$$V_e = 2.98 \times 10^6 \text{ cm s}^{-1}$$

"Best case" collision velocity $|v - v_e| = 1.23 \times 10^6 \text{ cm s}^{-1}$

"Worst case" " " $|v + v_e| = 7.19 \times 10^6 \text{ cm s}^{-1}$

(d) Condition for neglecting the potential of the Earth is that the collision velocity at large distance be large compared to the Earth escape velocity.

$$V_{esc} = \sqrt{\frac{2GM_\oplus}{R_\oplus}} \simeq 1.12 \times 10^6 \text{ cm s}^{-1}$$

$\Rightarrow$ neglect of the acc^n is justified in the "worst case" scenario, but not in the "best case"

(e) Total kinetic energy $E_{KE} = \frac{1}{2} M v^2$

$$M = \frac{4}{3} \pi S^3 \rho_M = 5.24 \times 10^{17} \text{ g.}$$

$$\Rightarrow \quad E_{KE,\,best} = 3.96 \times 10^{29} \text{ erg}$$

$$E_{KE,\,worst} = 1.35 \times 10^{31} \text{ erg}$$

(f) In megatons:

$$E_{KE,\,best} \simeq 9 \times 10^6 \text{ MT}$$

$$E_{KE,\,worst} \simeq 3 \times 10^8 \text{ MT}$$

$\therefore$ even the best case scenario is pretty bad!

(g) Using the same eq^n for part (b), velocity at distance of Jupiter (5.2AU) is:

$$v \simeq 1.8 \times 10^6 \text{ cm s}^{-1}$$

This velocity is mostly radial. For an estimate, assume constant velocity and an additional distance of $\sim$ 5AU to travel:

$$t \sim \frac{d}{v} \sim 1.3 \text{ years.}$$

$\Rightarrow$ we expect about a year's advance warning.